

Remarks on Strong CP -Violating Lagrangians

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Abstract

Owing to a different treatment of the vacuum alignment, the strong CP -violating Lagrangian obtained by Di Vecchia, Veneziano and Witten (DVW) 3 decades ago do not look quite the same as the one originally derived by Baluni at the quark or hadron level. We show that they are consistent with each other and emphasize that, within the DVW approach, the $\theta G\tilde{G}$ term is not entirely removed away after the vacuum is rotated from the CP -odd state to the CP -even one; strong CP violation resides not only in the quark mass terms but also in the residual topological sector. Contrary to some claims, it is necessary to include the $SU(3)$ -singlet η_0 tadpole contribution for strong CP -odd effects induced by the Baluni-type Lagrangian to ensure that strong CP violation vanishes in the zero axial anomaly limit.

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1. The commonly used strong CP -violating quark operator first derived by Baluni [1] and the chiral Lagrangian derived by Di Vecchia, Veneziano [2] and by Witten [3] (DVW) 3 decades ago do not look quite the same. Especially, the treatment of the vacuum alignment seems to be quite different. Whether these CP -odd Lagrangians are equivalent is the issue to be explored in this short note.

It is well known that the nontrivial topological structure of the θ vacuum in QCD not only allows for instanton solutions but also induces an additional T - and P -violating θ term to the QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}GG + \sum_i \bar{q}_i(i\not{D} - m_i)q_i - \frac{g^2}{32\pi^2}\theta_{\text{QCD}}G\tilde{G}. \quad (1)$$

The θ vacuum is generally P and CP noninvariant. In practice, it is more convenient to work with the CP -invariant vacuum so that the $\theta_{\text{QCD}}G\tilde{G}$ term is represented as an operator perturbation and CP is explicitly broken. This can be achieved by going to a basis where $\theta_{\text{QCD}}G\tilde{G}$ is rotated away and replaced by an effective CP -odd operator in terms of quark fields. Owing to the axial anomaly, a chiral rotation of the quark field $q \rightarrow \exp(i\alpha\gamma_5)q$ will induce a change of θ_{QCD} , namely, $\theta_{\text{QCD}} \rightarrow \theta_{\text{QCD}} - 2\alpha$.

In general, the vacuum expectation value (VEV) of the quark condensate has the expression

$$\langle \bar{q}_{iL}q_{jR} \rangle = -\Lambda^3 \delta_{ij} e^{-i\phi_i}. \quad (2)$$

P and CP symmetries require that $\phi_i = 0, \pi$. We now make a chiral rotation of the quark field $q_{iR} \rightarrow \exp(i\phi_i/2)q_{iR}$ so that $\langle \bar{q}_{iL}q_{jR} \rangle = -\Lambda^3 \delta_{ij}$. The QCD Lagrangian then reads

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}GG + \sum_i \bar{q}_i i\not{D} q_i - \sum_i m_i \cos \phi_i \bar{q}_i q_i + \sum_i m_i \sin \phi_i \bar{q}_i i\gamma_5 q_i - (\theta - \sum \phi_i)q(x), \quad (3)$$

with $q(x) \equiv (g^2/32\pi^2)G\tilde{G}$, where for simplicity we have dropped the subscript “QCD” of θ . To avoid the vacuum instability, the phases ϕ_i should satisfy the constraints [6, 7]

$$m_i \sin \phi_i = m_j \sin \phi_j, \quad \sum_i \phi_i = \theta. \quad (4)$$

As we shall see below, the first constraint arises from the vacuum alignment, while the second one comes from the anomalous Ward-Takahashi (WT) identity. Eqs. (3) and (4) lead to the well known strong CP -violating operator first derived by Baluni [1]:

$$\delta\mathcal{L}_{CP}^{\text{Baluni}} = \theta \bar{m}(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d + \bar{s}i\gamma_5 s), \quad \bar{m} = (1/m_u + 1/m_d + 1/m_s) \quad (5)$$

for $\theta \ll 1$ and three light quarks. In principle, one can apply current algebra to compute the hadronic matrix elements induced by $\delta\mathcal{L}_{CP}^{\text{Baluni}}$.¹ However, it is much more convenient in practice

¹ In lattice or quark model calculations, it is the disconnected insertion of $\delta\mathcal{L}_{CP}^{\text{Baluni}}$ that is related to the insertion of $\theta G\tilde{G}$ in the hadronic process [4]. The connected insertion of $\delta\mathcal{L}_{CP}^{\text{Baluni}}$ must vanish for on-shell amplitudes. Since a quark loop with the insertion of $\delta\mathcal{L}_{CP}^{\text{Baluni}}$ at zero momentum is the same as an insertion of $\theta G\tilde{G}$, the trick of rotating the θ term into a pseudoscalar density does not change the nature of the calculation which must be done in the lattice or quark model framework [4]. Hence, if care is not taken, the use of Baluni’s Lagrangian may lead to fake results, for example, the electric dipole moment of the constituent quark [5].

to use $U(3)$ chiral perturbation theory valid in the large N_c limit to get the chiral representation for the pseudoscalar quark density:

$$\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d + \bar{s}i\gamma_5 s = -\frac{i}{4}f_\pi^2 v \text{Tr}(U - U^\dagger). \quad (6)$$

with $U = \exp(2i\phi/f_\pi)$, $\phi = \sum_{a=0}^8 \phi^a \lambda^a / \sqrt{2}$, $\text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$, $f_\pi = 132$ MeV, and

$$v = -2 \frac{\langle \bar{q}q \rangle}{f_\pi^2} \quad (7)$$

characterizing the spontaneous breaking of chiral symmetry. Consequently, the chiral realization of $\delta\mathcal{L}_{CP}^{\text{Baluni}}$ in the meson sector is

$$\delta\mathcal{L}_{CP}^M = -\frac{i}{4}\theta\bar{m}f_\pi^2 v \text{Tr}(U - U^\dagger) = \sqrt{3}\theta\bar{m}f_\pi v\eta_0 + O(\phi^3) + \dots \quad (8)$$

Using the vacuum expectation value

$$\langle q(x) \rangle = \frac{1}{8} \frac{af_\pi^2}{N_c} (\theta - \sum_i \phi_i) \quad (9)$$

derived from the chiral Lagrangian approach (see below) with the parameter a denoting the mass squared of η_0 in the chiral limit, Di Vecchia and Veneziano [2] and Witten [3] obtained a different energy minimizing condition

$$m_i \sin \phi_i = \frac{a}{2vN_c} (\theta - \sum_i \phi_i). \quad (10)$$

Hence, in QCD language one has (see Eq. (A.14) of [2])

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}GG + \sum_i \bar{q}_i i \not{D} q_i - \sum_i m_i \cos \phi_i \bar{q}_i q_i - (\theta - \sum_i \phi_i) \left(q(x) - \frac{a}{2vN_c} \sum_j \bar{q}_j i\gamma_5 q_j \right). \quad (11)$$

The chiral realization of the last term is

$$\delta\mathcal{L}_{CP}^{\text{DVW}} = -i \frac{af_\pi^2}{8N_c} \bar{\theta} \left(\text{Tr}(U - U^\dagger) - \text{Tr}(\ln U / U^\dagger) \right) \quad (12)$$

valid to the leading order of $\bar{\theta} \equiv \theta - \sum \phi_i$. As stressed by Di Vecchia, Veneziano and Witten, an important feature of this CP -violating interaction is that it does not contain terms linear in any of the pseudoscalar fields including the flavor-singlet η_0 .

A comparison of the last term of Eq. (11) with $\delta\mathcal{L}_{CP}^{\text{Baluni}}$ or $\delta\mathcal{L}_{CP}^{\text{DVW}}$ with $\delta\mathcal{L}_{CP}^M$ reveals that they are not obviously equivalent. First, it is evident from Eq. (11) that, at the quark level, if the $\theta q(x)$ term is rotated away with $\sum \phi_i = \theta$, there will be no strong CP violation at all: A disappearance of strong CP nonconservation in the topological gauge sector will not resurrect in the quark mass sector. Second, while the η_0 is eliminated from $\delta\mathcal{L}_{CP}^{\text{DVW}}$, it is not so in $\delta\mathcal{L}_{CP}^M$. Indeed, it has been pointed out in [8] and will be stressed again below with an explicit example that for strong CP -violating effects induced by $\delta\mathcal{L}_{CP}^M$, it is necessary to include the η_0 tadpole contribution to ensure that the physical effect vanishes in the zero anomaly limit. Since the CP -odd Lagrangian $\delta\mathcal{L}_{CP}^{\text{DVW}}$ has been popularly used in the literature (see e.g. [9]), the purpose of this short note is to clarify the aforementioned issues.

2. Following Crewther [6, 7] we begin with a re-derivation of the first constraint in Eq. (4). It is known that the explicit chiral symmetry breaking terms by quark masses provide a preferred

direction for spontaneous chiral symmetry breaking. When explicit and spontaneous symmetry breaking are badly misaligned, explicit symmetry breaking cannot be treated as a small perturbation (for a recent discussion, see [10]). To find this direction one can apply the Dashen's theorem [11] which states that the VEV

$$\langle U(\vec{\omega})(-\mathcal{L}_{\text{mass}})U(\vec{\omega})^{-1} \rangle \quad (13)$$

has a minimum at $\vec{\omega} = 0$, where $U(\vec{\omega}) = \exp(i\vec{\omega} \cdot \vec{F})$ with \vec{F} being the generators of $SU(n) \times SU(n)$ and $\vec{\omega} \ll 1$. It suffices to consider the right-handed $SU(n)$ rotations which amount to replacing the phases ϕ_i by $\phi_i + \omega_i$, subject to the $SU(n)$ constraint $\sum_{i=1}^n \omega_i = 0$. Hence,

$$\langle -\mathcal{L}_{\text{mass}} \rangle_{\vec{\omega}} = -\frac{1}{2}f_\pi^2 v \sum_i m_i \cos(\phi_i + \omega_i) \quad (14)$$

should have a local minimum at $\vec{\omega} = 0$ where use of $\langle \bar{q}i\gamma_5 q \rangle = 0$ has been made. Note the term $(\theta - \sum \phi_i)q(x)$ in Eq. (3) is irrelevant for our purpose because of the flavor-blind nature of $q(x)$ and the constraint $\sum \omega_i = 0$. As pointed out by Crewther [6], instead of using the Lagrangian multiplier method elucidated by Nuyts [12], it is sufficient to consider the variation $\vec{\omega}$ as the linear combination of the variations

$$\omega_i = -\omega_j = \omega, \quad \omega_k = 0, \quad (k \neq i, j). \quad (15)$$

It follows from $d\langle -\mathcal{L}_{\text{mass}} \rangle_{\vec{\omega}}/d\omega = 0$ that

$$m_i \sin \phi_i = m_j \sin \phi_j = \lambda + \mathcal{O}(\epsilon^2) \quad (16)$$

is independent of the flavor index i and valid to the first order in chiral symmetry breaking (usually denoted by the parameter $\epsilon \propto m_q/m_N$ which approaches to zero in the chiral limit). A second constraint comes from the anomalous WT identity which leads to

$$\sum_{i=1}^n \frac{\partial \phi_i}{\partial \theta} - 1 = \mathcal{O}(\epsilon). \quad (17)$$

For the explicit expression of the $\mathcal{O}(\epsilon)$ terms on the right hand side of the above equation, see [6, 7] for details. This leads to

$$\sum_i \phi_i = \theta + \mathcal{O}(\epsilon), \quad (18)$$

which is the second constraint in Eq. (4) where the integration constant is fixed by requiring $\phi_i = 0$ to correspond to $\theta = 0$. For small θ and ϕ_i , it follows from Eq. (4) that

$$\lambda = \bar{m}\theta + \mathcal{O}(\epsilon^2, \theta^2). \quad (19)$$

A different minimization procedure was considered by Di Vecchia, Veneziano and Witten. The vacuum expectation value of the potential energy corresponding to \mathcal{L}_{QCD} in Eq. (3) is given by

$$\langle V(\phi_i) \rangle = -\frac{1}{2}f_\pi^2 v \sum_i m_i \cos \phi_i + \frac{1}{8} \frac{af_\pi^2}{N_c} (\theta - \sum_i \phi_i)^2, \quad (20)$$

where we have used Eqs. (7) and (9). The minimization condition $\partial \langle V(\phi_i) \rangle / \partial \phi_i = 0$ leads to Eq. (10).

The two approaches for the vacuum alignment look quite different. The first approach due to Baluni [1] and Crewther [6] relies on the Dashen's theorem where $q(x)$ plays no role, while the second one requires the external information on the ϕ_i dependence of $q(x)$ which can be obtained from the chiral Lagrangian approach. Since Eq. (10) does not allow the solution $\sum \phi_i = \theta$ for $\theta \neq 0$ and $\phi_i \neq 0$, it appears naively that this is not consistent with the second constraint of (4). However, a non-vanishing $\bar{\theta} = \theta - \sum \phi_i$ of order ϵ is allowed by Eq. (18). For $\phi_i \ll 1$, it follows from Eq. (10) that

$$\frac{a}{N_c} \bar{\theta} = \theta \left(\sum_i \frac{1}{2m_i v} + \frac{N_c}{a} \right)^{-1} \approx 2\bar{m}v\theta, \quad (21)$$

where we have neglected the term N_c/a compared to $1/(2m_i v)$ as $a \approx 0.73 \text{ GeV}^2$ and $v \approx 1.6 \text{ GeV}$ numerically. Therefore, the minimization condition (10) obtained by Di Vecchia, Veneziano and Witten is equivalent to that of Eqs. (16) and (19). We find numerically $\bar{\theta} \approx 2 \times 10^{-2} \theta$. This means $\sum_i \phi_i$ is indeed very close to but not exactly identical to θ . The last term in the QCD Lagrangian (11) is identical to $\delta \mathcal{L}_{CP}^{\text{Baluni}}$ except that there is a residual contribution from the topological charge density $q(x)$ owing to the fact that $\bar{\theta} \neq 0$.² At the hadron level we can use the chiral Lagrangian approach to determine the chiral realization of the $G\tilde{G}$ term. Given the flavor-blind nature of $q(x)$, it is natural to argue that the flavor-singlet η_0 should be one of the interpolating fields for $G\tilde{G}$. For the case of $\sum_i \phi_i = \theta$, there is no strong CP violation. In this case, possible solutions to Eq. (10) such as $\theta = 0$ and all $\phi_i = 0$ have been discussed in [2, 3].

In short, a major difference between the two approaches is that the sum of the phases of the quark condensate $\sum \phi_i$ is taken to be the same as the θ parameter in the Baluni approach, while there is a small and calculable deviation of $\sum \phi_i$ from θ in the DVW scenario. As a consequence, the $\theta G\tilde{G}$ term in the DVW approach is not entirely removed away after the vacuum is rotated from the CP -odd state to the CP -even one; strong CP violation resides not only in the quark mass terms but also in the residual topological sector.

3. In the chiral-Lagrangian approach, the effective meson Lagrangian respected the anomalous WT identity in the leading $1/N_c$ expansion has the form [2, 13–15]

$$\begin{aligned} \mathcal{L}_M = & \frac{f_\pi^2}{8} [\text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + 2v \text{Tr}(mU^\dagger + mU)] + \frac{N_c}{2} \frac{1}{af_\pi^2} (\partial^\mu K_\mu)^2 \\ & + \frac{i}{4} (\partial^\mu K_\mu) (\ln \det U - \ln \det U^\dagger) - \theta \frac{g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}, \end{aligned} \quad (22)$$

where K_μ is the Chern-Simons current with the divergence $\partial^\mu K_\mu = (g^2/16\pi^2) G\tilde{G}$ and m is a diagonal quark mass matrix. Since $UU^\dagger = 1$, the vacuum expectation value of $U(x)$, $\langle U \rangle$, can be written as a diagonal matrix V with the matrix elements $(e^{-i\phi_u}, e^{-i\phi_d}, e^{-i\phi_s})$. Making a chiral

² In principle, one can make a chiral rotation of the quark field $q_i \rightarrow \exp(i\phi'_i \gamma_5/2) q_i$ to remove away the $\theta G\tilde{G}$ term completely from the QCD Lagrangian provided that $\sum_i \phi'_i = \theta$. However, a small deviation of ϕ'_i from ϕ_i , the phase of the quark condensate [see Eq. (2)], implies that the vacuum is not CP -even. As stressed in the beginning, we prefer to work with the CP -invariant vacuum so that the bulk of the $\theta_{\text{QCD}} G\tilde{G}$ term is represented as an operator perturbation and CP is explicitly broken.

rotation $U \rightarrow UV$ so that $\langle U \rangle = 1$ and

$$\begin{aligned} \mathcal{L}_M = & \frac{f_\pi^2}{8} [\text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + 2v \text{Tr}(mV^\dagger U^\dagger + mUV)] + \frac{N_c}{2} \frac{1}{af_\pi^2} (\partial^\mu K_\mu)^2 \\ & + \frac{i}{4} (\partial^\mu K_\mu) (\ln \det U - \ln \det U^\dagger) - \frac{1}{2} \bar{\theta} \partial^\mu K_\mu. \end{aligned} \quad (23)$$

The use of the equation of motion yields

$$\partial^\mu K_\mu = -\frac{i}{4} \frac{af_\pi^2}{N_c} (\ln \det U - \ln \det U^\dagger) + \frac{1}{2} \frac{af_\pi^2}{N_c} \bar{\theta} = \sqrt{3} \frac{af_\pi}{N_c} \eta_0 + \frac{1}{2} \frac{af_\pi^2}{N_c} \bar{\theta}. \quad (24)$$

This gives the chiral realization of the $\frac{1}{2}q(x)$ term. Putting this back to \mathcal{L}_M gives an additional mass term $-\frac{1}{2}(3a/N_c)\eta_0^2$ for the η_0 due to the axial anomaly. The VEV of $q(x)$ as shown in Eq. (9) also follows from the above equation.

It is straightforward to show that the use of the Dashen's theorem for the vacuum alignment will lead to the strong CP -violating operator $\delta\mathcal{L}_{CP}^M$, while the minimization condition $\langle \partial V(\phi_i) \rangle / \partial \phi_i = 0$ yields $\delta\mathcal{L}_{CP}^{\text{DVW}}$. Applying the relation $a\bar{\theta}/N_c = 2\bar{m}v\theta$ again, we see that $\delta\mathcal{L}_{CP}^{\text{DVW}}$ is the same as $\delta\mathcal{L}_{CP}^M$ except for an additional $\text{Tr}(\ln U/U^\dagger)$ term

$$\delta\mathcal{L}_{CP}^{\text{DVW}} \rightarrow \delta\mathcal{L}_{CP}^M = -\frac{i}{4} \theta \bar{m} f_\pi^2 v \left(\text{Tr}(U - U^\dagger) - \text{Tr}(\ln U/U^\dagger) \right). \quad (25)$$

This extra term comes from the residual $G\tilde{G}$ sector and is governed by the η_0 field. Because of the large mass of the η_0 , its tadpole contribution is very small in practice and hence can be neglected. In this sense, the Lagrangians $\delta\mathcal{L}_{CP}^{\text{DVW}}$ and $\delta\mathcal{L}_{CP}^M$ are equivalent. However, we would like to make a caveat here that this CP -odd operator $\delta\mathcal{L}_{CP}^M$ is not the right one for studying the zero axial anomaly behavior. This will be explained below.

4. A common feature of the strong CP -odd Lagrangians $\delta\mathcal{L}_{CP}^M$ and $\delta\mathcal{L}_{CP}^{\text{DVW}}$ is that they cannot create Goldstone bosons from the vacuum, $\langle 0 | \delta\mathcal{L}_{CP} | G^a \rangle = 0$ with $a = 1, \dots, 8$ [11].³ This is understandable because if Goldstone bosons can be created from $\delta\mathcal{L}_{CP}$, they will be produced so abundantly to bring a shift of the vacuum and cause a vacuum instability. However, the CP -odd operator $\delta\mathcal{L}_{CP}^{\text{DVW}}$ goes one step further by having $\langle 0 | \delta\mathcal{L}_{CP}^{\text{DVW}} | \eta_0 \rangle = 0$ even though the η_0 is not a Goldstone boson. As explained before, the decoupling of the η_0 from $\delta\mathcal{L}_{CP}^{\text{DVW}}$ is due to the residual $q(x)$ term which compensates precisely the η_0 field occurring in the CP -violating quark mass terms.

CP -violating effects induced by the QCD θ term must vanish in the chiral limit and/or in the absence of the axial anomaly [18]. Although the absence of strong CP violation in the zero anomaly limit is not obviously manifest in the strong CP -violating operator $\delta\mathcal{L}_{CP}^{\text{Baluni}}$ or $\delta\mathcal{L}_{CP}^M$, a correct evaluation of strong CP phenomena must respect the aforementioned constraints. By contrast, the advantage of $\delta\mathcal{L}_{CP}^{\text{DVW}}$ is that its induced matrix element vanishes obviously in the zero anomaly limit, though not manifestly in the chiral limit. In the following we take the CP -violating pion-nucleon coupling $\bar{g}_{\pi NN}$ induced by $\delta\mathcal{L}_{CP}^M$ as an example to point out that the tadpole contribution due to the η_0 must be taken into account in order to produce the correct $a \rightarrow 0$ behavior [8].

³ This condition can be used as the starting point for deriving the Baluni Lagrangian, see [17].

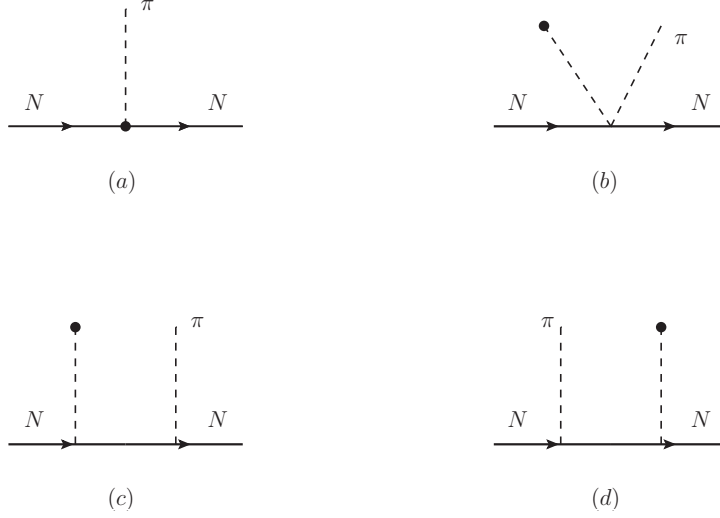


FIG. 1: Diagrams contributing to the CP -violating pion-nucleon coupling $\bar{g}_{\pi NN}$. The dark blob indicates a vertex induced by the CP -odd Lagrangian $\delta\mathcal{L}_{CP}^M$ or $\delta\mathcal{L}_{CP}^B$.

In the following we recapitulate the calculations in [8]. There are four diagrams contributing to $\bar{g}_{\pi NN}$ (Fig. 1). The strong CP -violating operator in the baryon sector [8, 9, 16]

$$\delta\mathcal{L}_{CP}^B = -2\theta\frac{\bar{m}}{f_\pi}[b\text{Tr}(\bar{B}B\phi) + c\text{Tr}(\bar{B}\phi B)], \quad (26)$$

with

$$b = \frac{2(m_\Sigma - m_N)}{2m_s - m_u - m_d}, \quad c = -\frac{2(m_\Xi - m_\Sigma)}{2m_s - m_u - m_d} \quad (27)$$

contributes to Fig. 1(a). Fig. 1(b) arises from the η_0 tadpole contribution. Figs. 1(c) and 1(d) do not contribute to $\bar{g}_{\pi NN}$ owing to the derivative pion-nucleon coupling. The result is [8]

$$\bar{g}_{\pi NN} = 2\sqrt{2}\theta\frac{\bar{m}}{f_\pi}\frac{m_\Xi - m_\Sigma}{2m_s - m_u - m_d}(1 - I), \quad (28)$$

where

$$I = \frac{m_\pi^2}{m_{\eta'}^2}\cos\phi\left(\cos\phi + \frac{1}{\sqrt{2}}\sin\phi\right) - \frac{m_\pi^2}{m_\eta^2}\sin\phi\left(\frac{1}{\sqrt{2}}\cos\phi - \sin\phi\right) \quad (29)$$

comes from the η_0 tadpole contribution and ϕ is the $\eta - \eta'$ mixing angle defined by

$$\eta = \eta_8 \cos\phi - \eta_0 \sin\phi, \quad \eta' = \eta_8 \sin\phi + \eta_0 \cos\phi. \quad (30)$$

The expression of $\bar{g}_{\pi NN}$ in Eq. (28) without the correction I is precisely the current-algebra result obtained in [17]. In the limit $a \rightarrow 0$, one has

$$m_\eta^2 = 2m_K^2 - m_\pi^2, \quad m_{\eta'}^2 = m_\pi^2, \quad \phi = \arctan(1/\sqrt{2}). \quad (31)$$

Hence, the η' is as light as the pion in the absence of the axial anomaly. This is the well-known $U_A(1)$ problem. Since $I \rightarrow 1$ in the $a \rightarrow 0$ limit, it is evident that $\bar{g}_{\pi NN}$ vanishes when the gluonic anomaly is turned off, as it should be. Therefore, although the η_0 pole contribution is very small

numerically, it ensures that the physical result has the correct behavior in chiral and vanishing anomaly limit.

Now returning back to Eq. (25), it is clear that the CP -odd operator $\delta\mathcal{L}'_{CP}^M$ does not have the right behavior in the $a \rightarrow 0$ limit, though it is perfect in the real world where a is large. Indeed, the relation $a\bar{\theta}/N_c = 2\bar{m}v\theta$ does not hold for $a \rightarrow 0$ at first place. This indicates that it is better to use either $\delta\mathcal{L}_{CP}^M$ or $\delta\mathcal{L}_{CP}^{DVW}$ rather than the hybrid one $\delta\mathcal{L}'_{CP}^M$.

5. From $\delta\mathcal{L}_{CP}^M$ or $\delta\mathcal{L}_{CP}^{DVW}$ it is easily seen that strong CP violation can manifest in the meson sector only if the number of the involved pseudoscalar mesons is odd. Therefore, strong CP -violating effects at low energies can only be seen in the decays such as $(\eta, \eta', G) \rightarrow 2\pi, 4\pi$, $G \rightarrow K\bar{K}, \eta\eta$, and $(\eta, \eta', G) \rightarrow \gamma\gamma$ via FF coupling with G being a pseudoscalar glueball. Unfortunately, strong CP -odd effects at low energies are always proportional to θ^2 and hence extremely small except the baryon's electric dipole moment which is proportional to θ . This is why the neutron electric dipole moment provides the most stringent limit on θ . Nevertheless, P - and CP -violating metastable domains could be formed in heavy ion collisions at RHIC owing to the presence of the extremely strong magnetic field produced in such a collision (see e.g. [19] for a recent experiment). This provides a very exciting avenue to probe strong CP -violating effects induced by the θ vacuum.

6. Whether the strong CP -violating Lagrangians (11) and $\delta\mathcal{L}_{CP}^{DVW}$ obtained by Di Vecchia, Veneziano and Witten 3 decades ago are equivalent to the one $\delta\mathcal{L}_{CP}^{\text{Baluni}}$ originally derived by Baluni at the quark or hadron level is studied in this work. A major difference between the two approaches is that the sum of the phases of the quark condensate is taken to be the same as the θ parameter (i.e. $\sum \phi_i = \theta$) in the latter, while there is a small and calculable deviation of $\sum \phi_i$ from θ in the former. As a consequence, the $\theta G\tilde{G}$ term in the DVW approach is not entirely removed away after the vacuum is rotated from the CP -odd state to the CP -even one; strong CP violation resides not only in the quark mass terms but also in the residual topological sector. Neglecting the η_0 tadpole contribution which is generally very small numerically, the DVW and Baluni Lagrangians are equivalent. Contrary to some claims, it is necessary to include the $SU(3)$ -singlet η_0 tadpole contribution for strong CP -odd effects induced by the Baluni-type Lagrangian to ensure that strong CP violation vanishes in the zero axial anomaly limit.

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- [1] V. Baluni, Phys. Rev. D **19**, 2227 (1979).
 - [2] P. Di Vecchia and G. Veneziano, Nucl. Phys. B **171**, 253 (1980).

- [3] E. Witten, *Annals Phys.* **128**, 363 (1980).
- [4] S. Aoki, A. Gocksch, A. V. Manohar and S. R. Sharpe, *Phys. Rev. Lett.* **65**, 1092 (1990).
- [5] A. Abada, J. Galand, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, *Phys. Lett. B* **256**, 508 (1991).
- [6] R. J. Crewther, *Chiral Properties of Quantum Chromodynamics*, in *Field Theoretical Methods in Particle Physics*, Kaiserslautern 1979, ed. W. Rühl, Vol. 55B, NATO Advanced Study Institutes Series (Plenum, New York, 1980) p. 529.
- [7] R. J. Crewther, *Phys. Lett. B* **93**, 75 (1980) [Erratum-ibid. *B* **95**, 461 (1980)].
- [8] H.Y. Cheng, *Phys. Rev. D* **44**, 166 (1991).
- [9] A. Pich and E. de Rafael, *Nucl. Phys. B* **367**, 313 (1991).
- [10] E. Mereghetti, W. H. Hockings and U. van Kolck, *Annals Phys.* **325**, 2363 (2010) [arXiv:1002.2391 [hep-ph]].
- [11] R. Dashen, *Phys. Rev. D* **3**, 1879 (1971).
- [12] J. Nuyts, *Phys. Rev. Lett.* **26**, 1604 (1971).
- [13] C. Rosenzweig, J. Schechter and C. G. Trahern, *Phys. Rev. D* **21**, 3388 (1980).
- [14] K. Kawarabayashi and N. Ohta, *Nucl. Phys. B* **175**, 477 (1980).
- [15] P. Nath and R. L. Arnowitt, *Phys. Rev. D* **23**, 473 (1981).
- [16] P. Cho, *Phys. Rev. D* **48**, 3304 (1993); B. Borasoy, *Phys. Rev. D* **61**, 114017 (2000).
- [17] R.J. Crewther, P. Di Vecchia, G. Veneziano, and E. Witten, *Phys. Lett. B* **88**, 123 (1979); **91**, 487(E) (1980).
- [18] S. Aoki and T. Hatsuda, *Phys. Rev. D* **45**, 2427 (1992).
- [19] B. I. Abelev *et al.* [STAR Collaboration], *Phys. Rev. Lett.* **103**, 251601 (2009) [arXiv:0909.1739 [nucl-ex]].